ψ-onticity in the PBR theorem
A case study using the MKC nullification of the KS theorem

Philosophy of Physics Seminar
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Outline

- The PBR Theorem
- Two notions of ψ-onticity
- The KS Theorem
- The MKC models
- ψ-onticity of the MKC models

The PBR Theorem

Formal statement

Any ontic model that reproduces the predictions of QM and satisfies the Preparation Independence Postulate is ψ-ontic.

As a slogan

The quantum state is real.

Methodology

PBR Theorem

Any ontic model that reproduces the predictions of QM and satisfies the Preparation Independence Postulate is ψ-ontic.

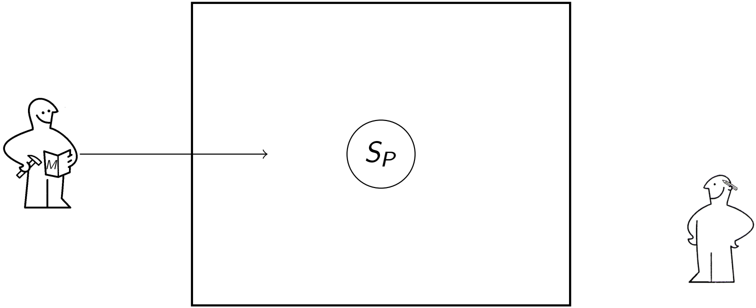
- Quantum mechanics is viewed as an *operational theory*.
  - Quantum states are viewed as *preparation procedures*.
  - Quantum observables are viewed as *measurement procedures*.

The operational approach

Operational Prepare Measurement model $(\mathcal{P}, \mathcal{M})$

$P \in \mathcal{P}$ *is a preparation,* $M \in \mathcal{M}$ *is a measurement,*

$\mathbb{P}(m|M,P)$ *probability of outcome m for measurement M performed on a system prepared according to P.*



Methodology

PBR Theorem

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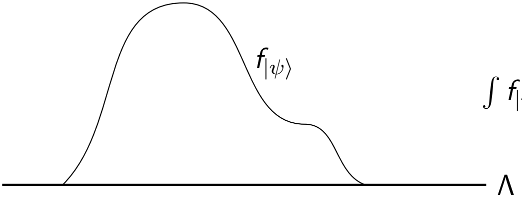
- Quantum mechanics is viewed as an *operational theory*.
  - Quantum states are viewed as *preparation procedures*.
  - Quantum observables are viewed as *measurement procedures*.
- The ontic models framework is a general framework in which systems are assigned *states*.
  - State space Λ.
  - Preparations are identified with probability distributions over Λ.
  - Measurements are identified with response functions.

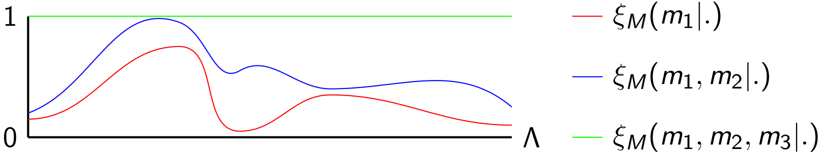
Ontic models

Preparation of $|\psi\rangle$:



$$\int f_{|\psi\rangle}(\lambda)\mathrm{d}\lambda = 1$$

Measurement of M:



$$\sum_i \xi_M(m_i|\lambda) = 1$$

Compatibility: $\mathbb{P}(m|M, P) = \int \xi_M(m|\lambda) f_{|\psi\rangle}(\lambda)\mathrm{d}\lambda.$

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- The ontic state λ determines the quantum state $|\psi\rangle$.

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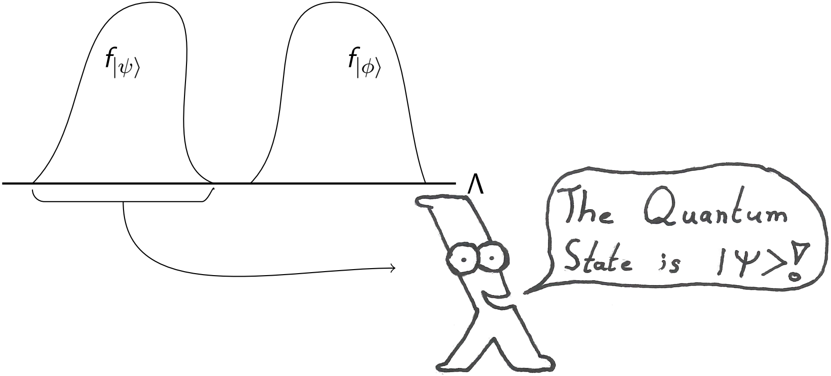
> *An important step towards the derivation of our result is the idea that the quantum state is physical if distinct quantum states correspond to non-overlapping distributions for λ.*



ψ-onticity

The ontic state λ determines the quantum state $|\psi\rangle$:

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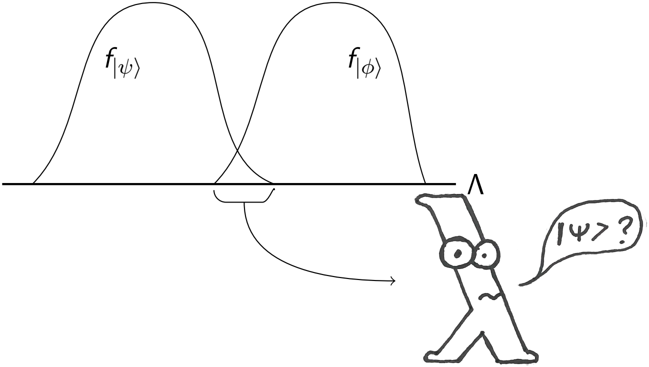

Methodology

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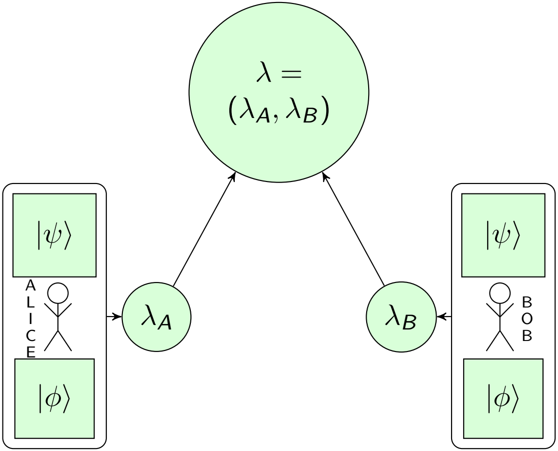
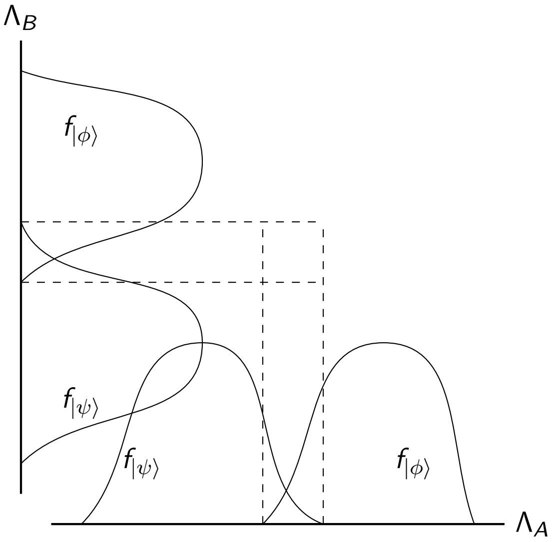
Preparation Independence



Preparation Independence



Without PIP

PBR Theorem

Any ontic model that reproduces the predictions of QM and satisfies the Preparation Independence Postulate is ψ-ontic.

BCLM Theorem

Any ontic model that reproduces the predictions of QM is almost ψ-ontic.

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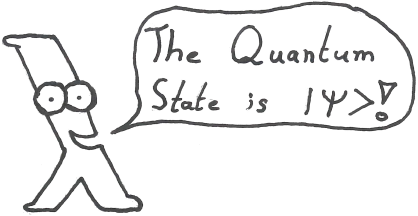
ψ-ontic Ontic Models

The ontic state tells us what the quantum state is



- There exists a function $f : \Lambda \to \mathcal{P}$

  $f(\lambda)$ is the true quantum state of the system,
- Compatibility with quantum mechanical notion of quantum states:

  $\mu_\psi(f^{-1}(|\psi\rangle)) = 1.$

ψ-ontic Ontic Models

1. $\exists f : \Lambda \to \mathcal{P}$ such that $\forall \left|\psi\right\rangle \ \mu_\psi(f^{-1}(\left|\psi\right\rangle)) = 1$.

Pusey, Barrett, Rudolph:

> *An important step towards the derivation of our result is the idea that the quantum state is physical if distinct quantum states correspond to non-overlapping distributions for* λ.

2. If $\left|\psi\right\rangle \neq \left|\phi\right\rangle$, then μ_ψ and μ_ϕ are non-overlapping.

Definition

An ontic model is ψ-ontic if for all $\left|\psi\right\rangle, \left|\phi\right\rangle \in \mathcal{P}$

$$D(\mu_\psi, \mu_\phi) = \sup_{\Omega \in \Sigma} |\mu_\psi(\Omega) - \mu_\phi(\Omega)| = 1.$$

Are ψ-ontic Ontic Models ψ-ontic?

1. $\exists f : \Lambda \to \mathcal{P}$ such that $\forall \left|\psi\right\rangle$ $\mu_\psi(f^{-1}(\left|\psi\right\rangle)) = 1$.
2. If $\left|\psi\right\rangle \neq \left|\phi\right\rangle$, then μ_ψ and μ_ϕ are non-overlapping.

Definition

An ontic model is ψ-ontic if for all $\left|\psi\right\rangle, \left|\phi\right\rangle \in \mathcal{P}$

$$D(\mu_\psi, \mu_\phi) = \sup_{\Omega \in \Sigma} |\mu_\psi(\Omega) - \mu_\phi(\Omega)| = 1.$$

- Clearly 1⇒2, but does 2⇒1?
- Intuitively: take $\Omega = \Lambda_\psi := f^{-1}(\left|\psi\right\rangle)$.
- But f need not exist for ψ-ontic models.

Case study: The MKC hidden variable models.

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Ontic Models and Hidden Variables

- Hidden variable models are traditionally concerned with the question:

  > Do measurements simply reveal the value of an observable, or is this value in some sense 'created' by the act of measurement?

- Ontic models do not provide an answer.

  $\xi(m|M,\lambda)$ provides a probability and λ does little to explain the transition from potential to actual.

- Determinate ontic states do give an answer:

$$\xi(m|M,\lambda) \in \{0,1\}.$$

The MKC models are constructed precisely to show the logical possibility of a **non-contextual hidden variable theory**. Allegedly, this possibility was ruled out by the Kochen-Specker theorem.

Assigning values to observables

Every non-degenerate self-adjoint operator A can be written as

$$A = a_1 \left|e_1\right\rangle \left\langle e_1\right| + \ldots + a_n \left|e_n\right\rangle \left\langle e_n\right|$$

with

$$\mathcal{B}_A = \{\left|e_1\right\rangle, \ldots, \left|e_n\right\rangle\}$$

an orthonormal basis.

- Assigning value a_j to A
- $=$ select vector $\left|e_j\right\rangle$ from $\mathcal{B}_A$.

For every non-degenerate self-adjoint operator A and every function f, the observables $f(A)$ and A can be measured jointly and the outcome for $f(A)$ is $f(a_j)$.

- The definite value assigned to $f(A)$ is determined by the definite value assigned to A.

The Kochen-Specker Theorem

Assigning definite values

An ontic state assigns definite values to observables by selecting for every orthonormal basis $\mathcal{B}$ the "true" vector.

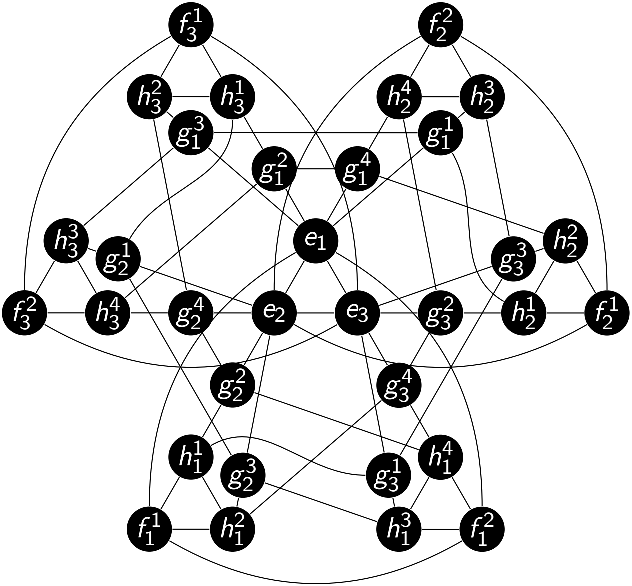
Non-Contextuality

For every ontic state: $|e\rangle$ is the "true" vector in an orthonormal basis $\mathcal{B}$ iff it is the true vector in every other orthonormal basis $\mathcal{B}'$ that contains $|e\rangle$.

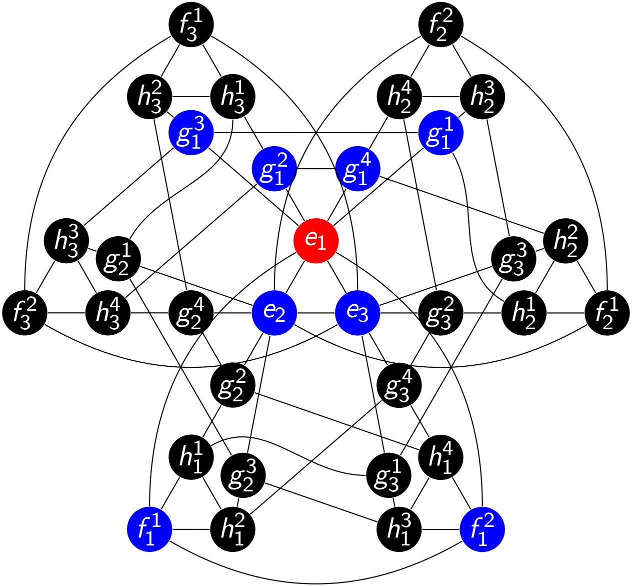
Kochen-Specker Theorem

There is a finite set of orthonormal bases for which one cannot select true vectors in a non-contextual way.

Uncolorable graph in 3 dimensions (Peres-cube)



Uncolorable graph in 3 dimensions



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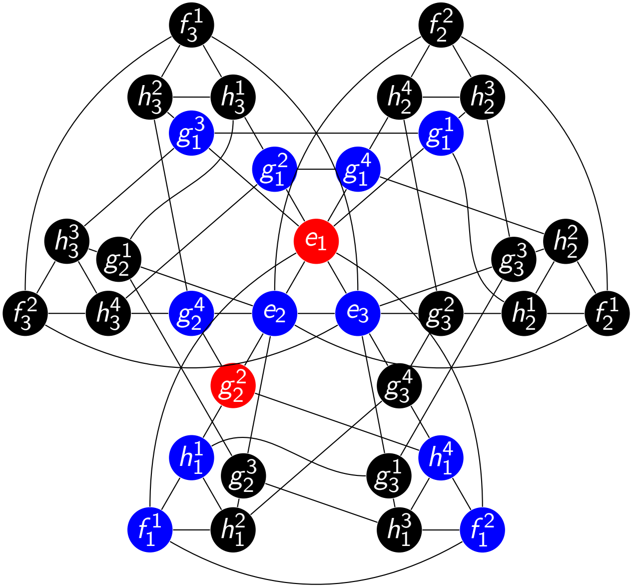

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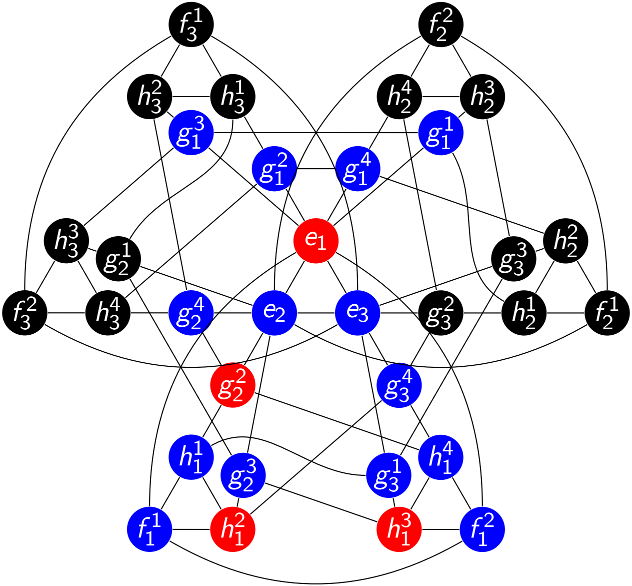

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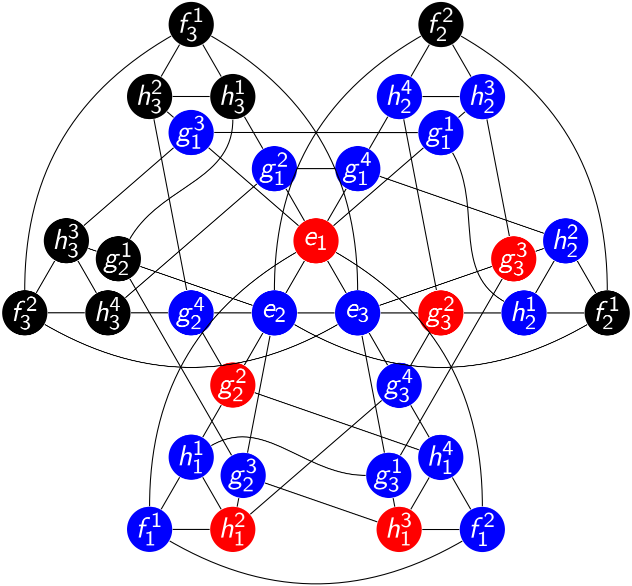

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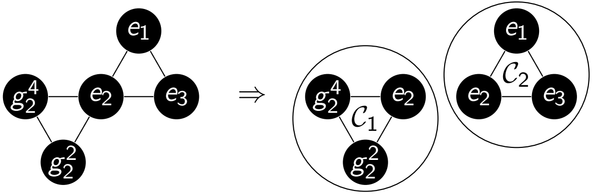
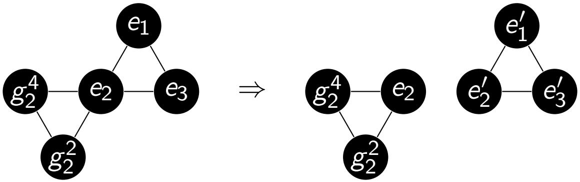
Circumventing the Kochen-Specker Theorem

Bell: Value assigned to an observable depends on the context $\mathcal{C}$



MKC: Not every orthonormal basis represents an observable



but every orthonormal basis can be approximated by an observable

$$0 < \|e_2 - e_2'\| < \epsilon.$$

The ontic models of Meyer, Kent and Clifton (MKC)

Definition

Two orthonormal bases

$$\mathcal{B}_1 = \left\{ \left| e_1^1 \right\rangle, \ldots, \left| e_n^1 \right\rangle \right\}, \; \mathcal{B}_2 = \left\{ \left| e_1^2 \right\rangle, \ldots, \left| e_n^2 \right\rangle \right\}$$

are totally incompatible if $\forall i, j = 1, \ldots, n$:

$$0 < \left| \left\langle e_i^1 \middle| e_j^2 \right\rangle \right| < 1.$$

Trivial Theorem

Let $\mathfrak{B}$ be any set of pairwise totally incompatible orthonormal bases. The set of all self-adjoint operators with eigenvectors in one of the bases in $\mathfrak{B}$ is colorable.

Non-Trivial Theorem (Clifton & Kent)

There exists a countable set $\mathfrak{B}$ of pairwise totally incompatible orthonormal bases that lies dense in the set of all orthonormal bases.

The ontic models of Meyer, Kent and Clifton (MKC)

- Ontic states: $\Lambda = \{\lambda : \mathfrak{B} \to \{1, \dots, n\}\}$.
- $\Sigma = \sigma$-algebra generated by cylinder sets.
- $\mathcal{P}$ determined by Born rule + independence of observables.
- $|e_i^k\rangle \in \mathcal{B}_k$:

$$C_{e_i^k} := \{\lambda \in \Lambda \mid \lambda(k) = i\},$$
$$\mu_\psi(C_{e_i^k}) := |\langle\psi|e_i^k\rangle|^2.$$

- $|e_{i_1}^{k_1}\rangle \in \mathcal{B}_{k_1}$, $|e_{i_2}^{k_2}\rangle \in \mathcal{B}_{k_2}$, …, $|e_{i_n}^{k_n}\rangle \in \mathcal{B}_{k_n}$:

$$C_{e_{i_1}^{k_1},\dots,e_{i_n}^{k_n}} := \bigcap_{j=1}^{n} \{\lambda \in \Lambda \mid \lambda(k_j) = i_j\},$$
$$\mu_\psi(C_{e_{i_1}^{k_1},\dots,e_{i_n}^{k_n}}) := \prod_{j=1}^{n} |\langle\psi|e_j^k\rangle|^2.$$

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Are the MKC models ψ-ontic?

PBR Theorem

Any ontic model that reproduces the predictions of QM and satisfies the Preparation Independence Postulate is ψ-ontic.

The MKC models have never been properly defined for composite systems. But reasonable attempts violate PIP.

BCLM Theorem

Any ontic model that reproduces the predictions of QM is not maximally ψ-epistemic (but "almost" ψ-ontic).

Are the MKC models ψ-ontic?

ψ-onticness:

For all pairs of quantum states ψ, ϕ, for all corresponding probability measures $\mu \in \Delta_\psi$, $\nu \in \Delta_\phi$ in the ontic model, the variational distance $D(\mu,\nu) := \sup_{\Omega\in\Sigma} |\mu(\Omega) - \nu(\Omega)|$ equals 1.

What is $D(\mu_\psi, \mu_\phi)$ in the MKC models?

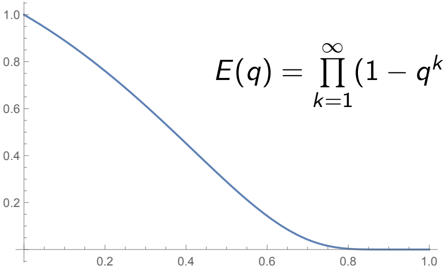
For cylinder sets:

$$\mu_\psi(C_{e_{i_1}^{k_1},\ldots,e_{i_n}^{k_n}}) := \prod_{j=1}^{n} |\langle\psi|e_{i_j}^{k_j}\rangle|^2 \to 1, \text{ as } n \to \infty,$$

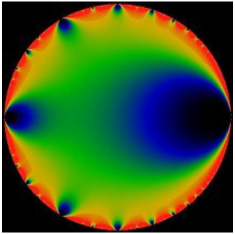
$$\mu_\phi(C_{e_{i_1}^{k_1},\ldots,e_{i_n}^{k_n}}) := \prod_{j=1}^{n} |\langle\phi|e_{i_j}^{k_j}\rangle|^2 \to 0, \text{ as } n \to \infty.$$

Finding ontic states for $|\psi\rangle$

Wanted: $\mu_\psi(C_{e_{i_1}^{k_1},\dots,e_{i_n}^{k_n}}) := \prod_{j=1}^n |\langle\psi|e_{i_j}^{k_j}\rangle|^2 \to 1,$ as $n \to \infty$,



$$E(q) = \prod_{k=1}^{\infty}(1 - q^k)$$

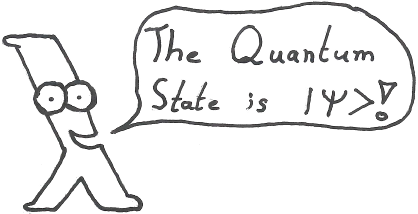


- Given $\epsilon > 0$, choose q_ϵ such that $E(x) > 1 - \epsilon$.
- For every k choose $(\mathcal{B}_{n_k}, |e_{i_k}^{n_k}\rangle)$ such that $|\langle\psi|e_{i_k}^{n_k}\rangle|^2 > 1 - q_\epsilon^k$.
- Set $\Lambda_\psi^\epsilon := \bigcap_{k=1}^\infty C_{e_{i_k}^{n_k}}$,

  then $\mu_\psi(\Lambda_\psi^\epsilon) > 1 - \epsilon$ and $\mu_\phi(\Lambda_\psi^\epsilon) = 0$ for all $|\phi\rangle$.

$\Rightarrow$ $D(\mu_\psi, \mu_\phi) = 1$

How ψ-ontic are the ψ-ontic MKC models?



1. There exists a function $f : \Lambda \to \mathcal{P}$, compatible with QM: $\mu_{|\psi\rangle}(f^{-1}(|\psi\rangle)) = 1$.
2. If $|\psi\rangle \neq |\phi\rangle$, then μ_ψ and μ_ϕ are non-overlapping.

- The MKC-models are ψ-ontic in the second sense: $D(\mu_\psi, \mu_\phi) = 1$.
- But we want the ontic state to tell us what $|\psi\rangle$ is, i.e., $\Lambda_\psi \subset \Lambda$ such that

$$\mu_\psi(\Lambda_\psi) = 1, \; \mu_\phi(\Lambda_\psi) = 0$$

How ψ-ontic are the ψ-ontic MKC models?

- Λ_ψ^ϵ does not contain all the ψ-ontic states: $\mu_\psi(\Lambda_\psi^\epsilon) < 1$.
- Moreover, there is no set Λ_ψ such that

$$\mu_\psi(\Lambda_\psi) = 1,\ \mu_\phi(\Lambda_\psi) = 0.$$

- Not all states in Λ_ψ^ϵ are ψ-ontic states:

$$\Lambda_\psi^\epsilon \cap \Lambda_\phi^\epsilon \neq \varnothing.$$

- The set Λ_ψ^ϵ is underdetermined by (ψ, ϵ).
- More generally: $f : \Lambda \to \mathcal{P}$ need not be unique, so we cannot speak of an identity relation.