Philosophy of quantum probability: An empiricist approach

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Motivation/Program

In the philosophy of probability there are two central questions we are concerned with. The first is: what is the correct formal theory of probability? [. . .] The second central question is: what do probability statements mean? – Lyon 2010

The formal theory of quantum probability is fairly odd.
Investigation may shed light on the meaning of probability statements in quantum mechanics. Perhaps overly optimistic:

Quantum theory is an evolution from Kolmogorov's probability theory rather than from Newtonian and Maxwellian physics – Cabello 2014

To understand quantum probability is to understand quantum theory!

Methodology

> *It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature. – Bohr 1927*

Ideally, what one *can* say is interpretation independent. And ideally, so is this contribution.

The following important questions are placed on hold:

- The measurement problem.
- The ψ-ontic/epistemic debate.
- The interpretation of probabilities in quantum mechanics.

Outline

- From classical to quantum probability.

  How does the formalism change?
- What are quantum probabilities probabilities of?

  Events, properties, and propositions.
- Reconstructing the quantum probability formalism.

  Quantum logic for empiricists.

Classical probability

The axioms of Kolmogorov (1933) for a *field of probability*

I. $\mathcal{F}$ is a field of subsets of a set Ω.

II. $\mathcal{F}$ contains the set Ω.

III. To each $A \in \mathcal{F}$ is assigned a non-negative real number $\mathbb{P}(A)$.

IV. $\mathbb{P}(\Omega) = 1$.

V. If A_1 and A_2 have no element in common, then $\mathbb{P}(A_1 \cup A_2) = \mathbb{P}(A_1) + \mathbb{P}(A_2)$.

Intended reading of the framework:

- Elements of $\mathcal{F}$ are (random) events.
- $\mathbb{P}(A)$ denotes the probability that A occurs.
- $A_1 \cap A_2$ represents the simultaneous occurrence of the events A_1 and A_2.
- If $A_1 \cap A_2 = \varnothing$, then the two events are incompatible.
- $A_1 \cup A_2$ represents the occurrence of at least one of the events A_1 and A_2.
- $A^c = \Omega \backslash A$ represents the non-occurrence of the event A.

Classical probability

Notes:

- Kolmogorov provides an axiomatization of probability, not an interpretation.
- The axioms are neither necessary nor sufficient conditions for something to be probability.
- 'Probability' is widely discussed in literature w.r.t. axioms III., IV. and V.
- 'Probability' is not the only primitive concept: the notion of 'event' also needs to be interpreted.
- 'Event' is often taken for granted together with axioms I. and II.
- But axioms I. and II. are rejected in quantum mechanics!

Events are sets?

In classical physics, events are the (revelation of) **properties** of a system.

- A system is (fully) characterized by a state ω.
- Ω denotes the set of all possible states.
- For every property A of the system, ω determines its value $\omega(A)$.
- The event [property A has value a] can then be identified with the set

$$\{\omega \in \Omega \mid \omega(A) = a\}.$$

The existence of properties is a metaphysical assumption.
Sufficient, but not necessary for the application of probability theory.
Example: the outcome of a coin toss.

Events are sets? II

Events can also be understood as **propositions**.

- For a formal language L, one can define an equivalence relation on sentences:

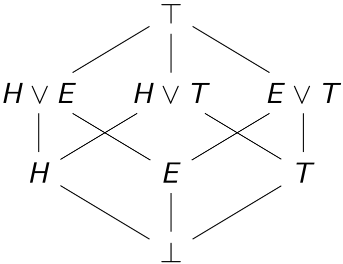
$$\phi \sim \psi \text{ if and only if } \vdash \phi \Leftrightarrow \psi$$

- The Lindenbaum-Tarski algebra is the collection of equivalence classes of sentences.
- When using classical logic, this is a Boolean algebra (Tarski 1935).
- Every Boolean algebra can be written as a field of subsets of some set (Stone 1936).
- This view is more general than the events as properties view. Property P can be replaced by the proposition "The system has property P".

Events are sets? III

Events are propositions, example.

Lindenbaum-Tarski algebra:



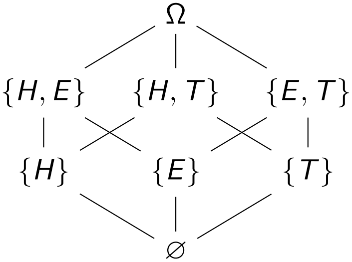
Elementary empirical sentences:

- H = "The coin lands heads."
- E = "The coin lands on the edge."
- T = "The coin lands tails."

Events are sets? III

Events are propositions, example.

Lindenbaum-Tarski algebra:



Elementary empirical sentences:

H = "The coin lands heads."

E = "The coin lands on the edge."

T = "The coin lands tails."

As an algebra of sets, this is the power set of $\Omega = \{H, E, T\}$.

Note: This is not a purely logical result. Empirical axioms play a role here too, such as

$$\vdash H \Rightarrow \neg T$$

Goal: To understand quantum probability using a similar logico-empirical approach.

Quantum probability

Axioms developed somewhere between 1932 (von Neumann) and 1992 (Parthasarathy):

I. $\mathcal{H}$ is a Hilbert space.
II. $L(\mathcal{H})$ is the set of all (closed) linear subspaces of $\mathcal{H}$.
III. To each $\mathcal{K} \in L(\mathcal{H})$ is assigned a non-negative real number $\mathbb{P}(\mathcal{K})$.
IV. $\mathbb{P}(\mathcal{H}) = 1$.
V. If $\mathcal{K}_1$ and $\mathcal{K}_2$ are orthogonal, then $\mathbb{P}(\mathcal{K}_1 \oplus \mathcal{K}_2) = \mathbb{P}(\mathcal{K}_1) + \mathbb{P}(\mathcal{K}_2)$.

Intended reading of the framework:

- Elements of $L(\mathcal{H})$ are (random) events.
- $\mathbb{P}(\mathcal{K})$ denotes the probability that $\mathcal{K}$ occurs.
- $\mathcal{K}_1 \cap \mathcal{K}_2$ represents the simultaneous occurrence of the events $\mathcal{K}_1$ and $\mathcal{K}_2$.
- If $\mathcal{K}_1 \cap \mathcal{K}_2 = \{0\}$, then the two events are incompatible.
- $\mathcal{K}_1 \oplus \mathcal{K}_2$ represents the occurrence of at least one of the events $\mathcal{K}_1$ and $\mathcal{K}_2$.
- $\mathcal{K}^\perp$ represents the non-occurrence of the event $\mathcal{K}$.

Events are subspaces?

- Classically, events are identified with sets.
- This could be motivated by either considering events to be properties or propositions.
- In quantum probability events are subspaces or projection operators, $L(\mathcal{H})$ is not a Boolean algebra.

What are quantum probabilities probabilities of?

Options:

(a) Subspaces can be understood as propositions.

Then some assumption leading to set-theory has to go,

or $L(\mathcal{H})$ does not give all the propositions (it is incomplete).

(b) Subspaces cannot be understood as propositions.

But possibly strongly related to propositions. One has to find this relation.

Could subspaces be propositions?

Birkhoff and von Neumann (1936): subspaces are mathematical representations of experimental propositions denoted "$A \in \Delta$".

A An observable.

Δ Subset of the set of possible measurement outcomes for A.

$\mathcal{K}_A^\Delta$ $= \{\psi \in \mathcal{H} \mid \mathbb{P}(A \in \Delta|\psi) = 1\}$ (following the Born rule) is the mathematical representation of $A \in \Delta$.

Every element of $L(\mathcal{H})$ can be obtained this way. Does this then provide the complete calculus of experimental propositions?
Two background assumptions can be identified for getting a "yes":

(a) It is unproblematic to 'forget the measurement context' when correlating an experimental proposition to the phase-space. ($\mathcal{K}_A^\Delta = \mathcal{K}_{A'}^{\Delta'}$ does not imply $A = A'$ and $\Delta = \Delta'$.)

(b) Disjunctions, conjunctions and negations of experimental propositions are again experimental propositions.

Could subspaces be propositions?

> *all well-defined experimental evidence, even if it cannot be analyzed in terms of classical physics, must be expressed in ordinary language – Bohr 1948*

If $A \in \Delta$ denotes an experimental proposition, then how can it be expressed in ordinary language? Suggestions:

$M_A(\Delta)$: "A is measured and the result lies in Δ".

$M_A \to \Delta$: "If A is measured, then the result lies in Δ".

Both run into difficulties with the non-distributivity of $L(\mathcal{H})$.

Example:

Let A, B be two 0,1-valued observables. Suppose $[A, B] \neq 0$ and that they (thus) cannot be measured simultaneously. Then quantum logic dictates:

$$\mathcal{K}_B^0 = \mathcal{K}_B^0 \wedge \left(\mathcal{K}_A^0 \vee \mathcal{K}_A^1\right) \neq \left(\mathcal{K}_B^0 \wedge \mathcal{K}_A^0\right) \vee \left(\mathcal{K}_B^0 \wedge \mathcal{K}_A^1\right) = \perp.$$

$\Longrightarrow$ At least one of the background assumptions has to go.

Reconstructing the quantum probability formalism

The two background assumptions:

(a) It is unproblematic to 'forget the measurement context' when correlating an experimental proposition to the phase-space. ($\mathcal{K}_A^{\Delta} = \mathcal{K}_{A'}^{\Delta'}$ does not imply $A = A'$ and $\Delta = \Delta'$.)

(b) Disjunctions, conjunctions and negations of experimental propositions are again experimental propositions.

Which one has to go and how to proceed?

Option 1: Think hard about the correct solution. ☹

Option 2: Start from scratch, build up a quantum logic of experimental propositions, and hope it is adequate. ☺

Reconstructing the quantum probability formalism II

Minimal requirements:

1) The formal language has to accommodate all elementary experimental propositions of the form

$M_A(\Delta)$: "A is measured and the result lies in Δ".

for every observable A and every (measurable) set of possible outcomes Δ.

2) Disjunctions, conjunctions and negations of elementary experimental propositions.

Like with the classical coin toss, additional empirical axioms are adopted:

- IEA (Idealized Experimenter Assumption): $M_A(\varnothing)$ implies $\perp$.
- LMR (Law Measurement Relation): If $A_2 = f(A_1)$, then $M_{A_1}(\Delta_1)$ implies $M_{A_2}(f(\Delta_1))$.
- NII (Non-commutativity Implies Incompatibility): If $[A_1, A_2] \neq 0$ then $M_{A_1}(\Delta_1) \wedge M_{A_2}(\Delta_2)$ implies $\perp$.

Reconstructing the quantum probability formalism III

The obtained quantum logic is

$$CQL := \mathcal{P}\left(\left\{(\mathcal{A}, P)\middle| \begin{matrix}\mathcal{A} \text{ unital Abelian algebra of operators on } \mathcal{H}\\ P \text{ an atom in the lattice of projectors in } \mathcal{A}\end{matrix}\right\}\right)$$

Cool Theorem: There is a set of conditions $C \subset CQL$ such that every quantum probability function $\mathbb{P} : L(\mathcal{H}) \to [0,1]$ can be represented as a conditional probability function $\mathbb{P} : CQL \times C \to [0,1]$.

$$\mathbb{P}(M_A(\Delta)|M_A) = \mathbb{P}(\mathcal{K}_A^{\Delta}).$$

Thus quantum probability spaces can be rewritten as Rényi-Popper spaces.

Not so cool: Not every Rényi-Popper function satisfies the Born rule. Solving this conundrum may require delving into metaphysical questions after all.

To be continued...

The end...

Thank You